

**Physical Dynamics (SPA5304) – Exercise Class**  
**Week 6 (17-Feb-2017)**

In all problems,  $G$  denotes the center of mass

**Problem 1**

A homogeneous ring of radius  $d$  is suspended from a point  $O'$  and can oscillate in a vertical plane. Find the equation(s) of motion and determine the frequency of small oscillations.

Repeat the exercise for a *homogeneous disk*. Will the disk oscillate at a higher or lower frequency?

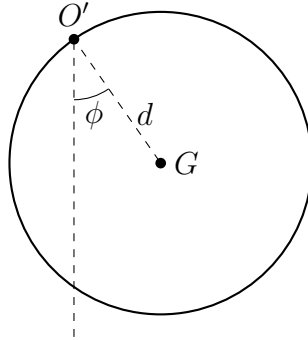


Figure 1: A homogeneous ring suspended by a point.

**Problem 2**

A homogeneous ring of mass  $m$  as seen in Figure 2 can roll on a surface without slipping. It is attached to a spring of elastic constant  $k$  and can oscillate on the horizontal plane. Find the Euler-Lagrange equation(s) and determine the frequency of small oscillations.

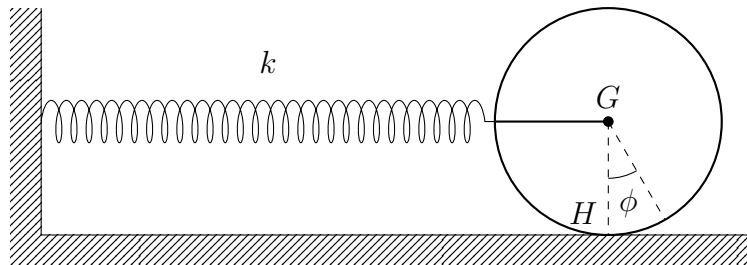


Figure 2: A homogeneous ring oscillating on a spring.

### Problem 3

A uniform rod of length  $2a$  has one end smoothly pivoted at a fixed point  $O$ . The other end is connected to a fixed point  $A$ , which is a distance  $2a$  vertically above the point  $O$ , by a light elastic spring of natural length  $a$  and elastic constant  $k = mg/2a$ . The rod moves in a vertical plane through  $O$ . Show that there are two equilibrium positions for the rod, and determine their stability. [The vertically upwards position for the rod would compress the spring to zero length and is excluded.]

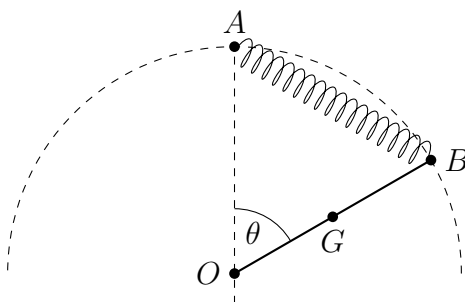


Figure 3: A homogeneous rod suspended on a spring.