Physical Dynamics (SPA5304) – Exercise Class
Week 5 (10-Feb-2017)

Problem 1

Use the Lagrangian formalism to find the equations of frictionless motion of the system of two blocks coupled with a spring. Find the energy E. Is it conserved? Does the system have any other conserved quantity (integral of motion)?

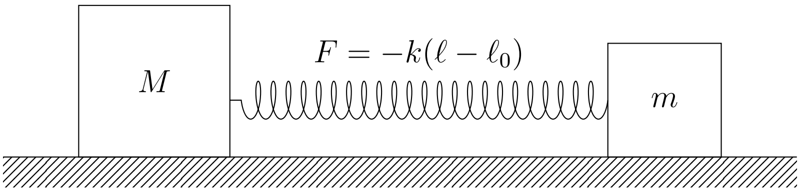

Figure 1: Two blocks coupled with a spring

Problem 2 – *Atwood's machine*

A uniform circular pulley of negligible mass can rotate freely about its axis of symmetry which is fixed in a horizontal position. Two masses m, $3m$ are connected by a light inextensible string which passes over the pulley without slipping. The whole system undergoes planar motion with the masses moving vertically. Take the rotation angle of the pulley as generalised coordinate and obtain Lagrange's equation for the motion. Deduce the acceleration of the mass m.

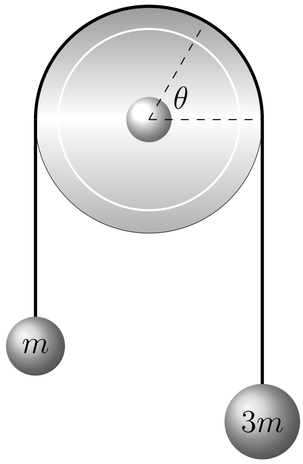

Figure 2: Atwood's machine

Problem 3

Consider the system in Figure 3. This is formed by two masses m and M connected by a massless string of fixed length ℓ, which hangs over two pulleys of negligible size. The left mass can only move along a vertical line, whilst the right mass can swing freely in the plane of the Figure. Gravity acts on the vertical direction as usual.

i. Choosing appropriate generalised coordinates, write down the Lagrangian and the Lagrange equations.

ii. Explain what are the forces acting on particle m, drawing them on the figure. Let $\vec{L}_m$ be the angular momentum of particle m with respect to the pulley on the right hand side. Calculate explicitly in terms of the generalised coordinates of the problems both sides of the equation derived in the lecture $\dot{\vec{L}}_{m,} = \vec{r} \times \vec{F}$, where $\vec{r}$ is the position vector of the particle m with respect to an origin in the pulley on the right hand side. Compare your result to the Lagrange equations obtained in point (i).

iii. Assume now that the right hand mass m can now only move vertically. Write down the Lagrange equations. The two masses are at rest at the initial time $t = 0$, and are at identical distance from the line connecting the pulleys (you can call d the distance between the pulleys). Assuming that $M > m$, determine at which time t^* the particle m will hit the pulley.

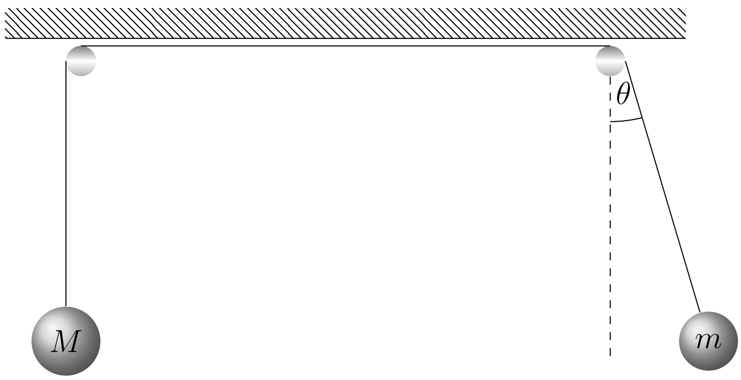

Figure 3: System of Problem 3