Physical Dynamics (SPA5304) – Exercise Class
Week 4 (3-Feb-2017)

Problem 1

A particle of mass m is constrained to move without friction on the surface of a cylinder of radius R in a gravitation field. Take as generalized coordinates the azimuthal angle φ and the vertical coordinate z. Write down the Lagrangian and the Euler-Lagrange equations. Show that φ is a cyclic coordinate and find (and identify) the conserved momentum p_ϕ.

Problem 2

Consider the system represented in Figure 1. The mass m_2 can move along the vertical axis, and the whole system rotates clockwise with constant angular velocity ω around this axis (you may find convenient to orient the z-axis as in the Figure). The length of the two rigid rods is equal to a, and their mass is negligible compared to m_1 and m_2. Gravity acts along the vertical axis.

i. How many degrees of freedom does the system have? Briefly explain your answer.
Hint: if your answer is 2, please reconsider it!

ii. Write the kinetic energy of the mass m_1 and of the mass m_2.

iii. Write the total potential energy for the system.

iv. Write the Lagrangian of the system.

v. Find the angular momenta $\vec{L}_1$ and $\vec{L}_2$ of m_1 and m_2, respectively, about the suspension point.

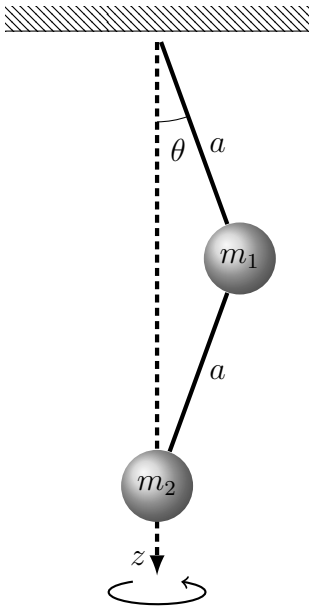

Figure 1: Rotating system of rods.

Problem 3

Consider the planar double pendulum in Figure 2.

i. How many degrees of freedom does the system have?

ii. Write the kinetic energy of the system.

iii. Write the total potential energy of the system.

iv. Write the Lagrangian of the system.

You may use as generalized coordinates the angles ϕ_1 and ϕ_2 (see Figure 2).

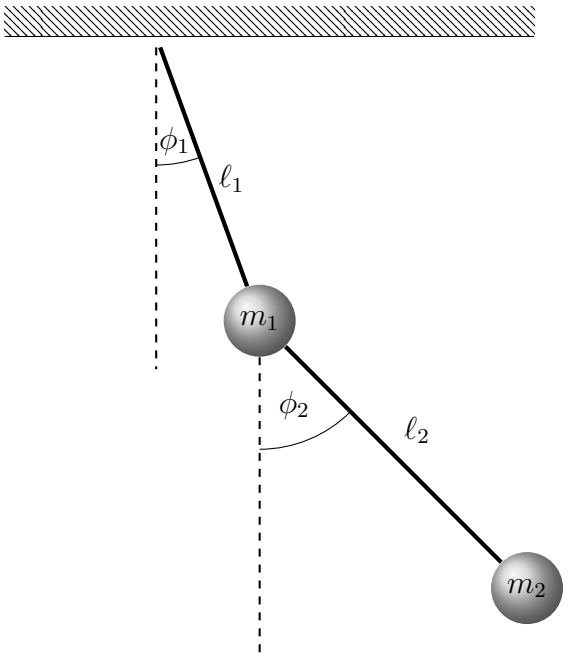

Figure 2: Double Pendulum.