Physical Dynamics (SPA5304) – Exercise Class
Week 3 (27-Jan-2017)

Problem 1

Consider a rocket lifting up against the Earth's uniform gravitational field. Let us denote the rocket's altitude above the launch pad at time t by $z(t)$, its speed by $v(t) = dz/dt$, and its mass by $M(t)$. The rocket's mass $M(t)$ changes in time because the rocket is emitting exhaust at speed u relative to it.

1. Show that the rocket's motion is described by the following equation:

$$\frac{dv}{dt} + \frac{u}{M(t)}\frac{dM(t)}{dt} = -g,$$

   where g is the acceleration due to gravity.

2. A particular rocket starts from rest at time $t = 0$ on the launch pad with a total take-off mass $M_0 = 10^5$ kg. The fuel has an exhaust speed $u = 2\ \mathrm{km\,s^{-1}}$. What is the minimum rate at which fuel must be burned (in kg $\mathrm{s^{-1}}$) in order to lift off the ground?

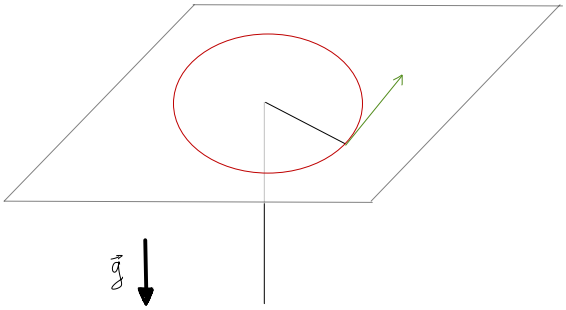
3. Assume that once the engines are started at $t = 0$, fuel is burned at a constant rate r, where $dM/dt = -r$, until the engines are switched off. Show that $z(t)$ satisfies the equation

$$\frac{d^2z}{dt^2} = -g + \frac{ur}{M_0 - rt}.$$

   Solve this and find $z(t)$.

Problem 2

A particle of mass m is attached to the end of a light string of length ℓ. The other end of the string is passed through a small hole and is slowly pulled through it. Gravity is negligible. The particle is originally spinning around the hole with angular velocity ω.



1. What are the forces acting on the particle of mass m? *(draw them on the figure)*
2. Show that the angular momentum of the particle with respect to the position of the hole is a conserved quantity.
3. Using the previous result, prove that the angular velocity becomes $4\,\omega$ when the string length is reduced to $\ell/2$.
4. Prove that the tension in the string when its length has the generic value r is equal to $m\,\omega^2\,\ell^4/r^3$.