Physical Dynamics (SPA5304) – Exercise Class
Week 1 (13/1/2017)

1. Consider the vectors

$$\vec{A} = (-3, 1, 2), \qquad \vec{B} = (-1, 2, 2).$$

   Compute:

   (a) $\|\vec{A}\|$ and $\|\vec{B}\|$

   (b) $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$

   (c) $\vec{A} \cdot \vec{B}$

   (d) $\vec{A} \times \vec{B}$

   (e) Let θ be the angle between $\vec{A}$ and $\vec{B}$. Using the results from (c) and (d), show that $\sin^2\theta + \cos^2\theta = 1$ as expected.

   (f) Show that $\vec{A} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{A} \times \vec{B}) = 0$. Why is that so?

2. In the following cases, compute $\dfrac{df}{dt}$ where $x = x(t)$

   (a) $f(x,t) = x^n$

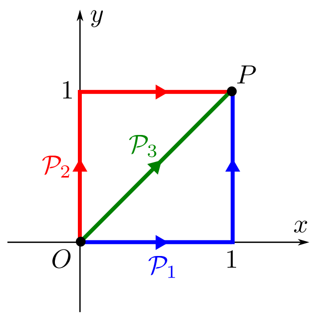
   (b) $f(x,t) = xe^x + 2t$

   (c) $f(x,t) = \sin^2(2x)$

3. (a) Assume that a particle experiences a force $\vec{F}$ that depends on the position $\vec{r} = (x, y, z)$. When $\vec{F}$ is given by

$$\vec{F}(\vec{r}) = (y^2, 2xy, 0),$$

   compute the work done by force $\vec{F}$ while the particle is moved inside the x-y plane (where $z = 0$) from point O to point P along three different paths $\mathcal{P}_1$, $\mathcal{P}_2$, $\mathcal{P}_3$ given below:



   Does the result suggest that the force is conservative or not?

(b) Compute the work when the force is given by

$$\vec{F}(\vec{r}) = (y, xy^2, 0)$$

for paths $\mathcal{P}_1$ and $\mathcal{P}_2$. Does the result suggest that the force is conservative or not?

(c) If force $\vec{F}$ is conservative, it is given in terms of a potential $V(\vec{r})$ as $\vec{F} = -\vec{\nabla}V$. Show that a conservative force $\vec{F}$ satisfies

$$\vec{\nabla} \times \vec{F} = 0.$$

This is a necessary condition for the force $\vec{F}$ to be conservative. (Actually, in normal situations this is a sufficient condition as well.)

(d) For the forces $\vec{F}$ given in (a) and (b), check if the above condition is satisfied or not. When it is satisfied, find the potential that gives the force.

4. A particle with mass m is connected to a spring and allowed to move in the x direction. If the equilibrium position is at $x = 0$, the elastic force acting on the particle is $F = -kx$ where k is the spring constant.

(a) Show that the potential is $V = \frac{k}{2}x^2$.

(b) Derive the equation of motion and rewrite it in terms of $\omega \equiv \sqrt{\frac{k}{m}}$.

(c) Integrate the equations of motion and find $x(t)$ explicitly in terms of the initial values $x(0), \dot{x}(0)$.

(d) Show explicitly that the total energy $E = T+V$ is constant in time where $T = \frac{m}{2}\dot{x}^2$ is the kinetic energy.