

With effect from 2004 the format of Third Year exam papers has changed. Students must answer **EVERY** question from section A and **TWO** questions from section B. You may assume that section B questions will be broadly similar questions from previous years, but here are some sample section A questions to show roughly what you can expect.

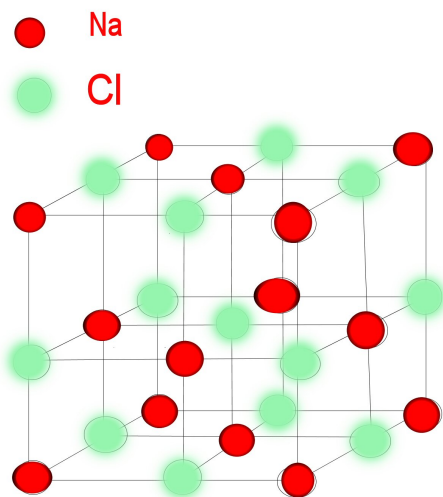
The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

SECTION A

[Part marks]

1. A *lattice* is an arrangement of points in space such that the environment of any point is identical with that of any other point. The *basis* is the arrangement of atoms associated with each point to generate the crystal.

The unit cell of sodium chloride is sketched below:



This is a face-centred cubic lattice, with a basis of Na at $(0, 0, 0)$ and Cl at $\frac{a}{2}(0, 0, 1)$ (there are various equivalent bases).

[5]

If the nearest-neighbour separation of ions is d , then the electrostatic energy per ion pair is $U = -e^2/(4\pi\epsilon_0 d)$. With $d = 0.28$ nm this gives

$$U = \frac{(1.6 \times 10^{-19})^2}{4\pi \times 8.854 \times 10^{-12} \times 0.28 \times 10^{-9}} = 8.2 \times 10^{-19} \text{ J}$$

or about 5.1 eV.

[2]

2. The separation of the (hkl) planes in a cubic crystal with lattice parameter a is $d_{hkl} = a/\sqrt{h^2 + k^2 + l^2}$. [2]

To analyse the x-ray diffraction experiment on Rb_3C_{60} we can draw up a table as follows (noting that the angle of deflection which we are given is twice the Bragg angle θ and that Bragg's law $2d \sin \theta = n\lambda$ may be rewritten in the form $(\sin \theta)^2 = \lambda^2(h^2 + k^2 + l^2)/(4a^2)$ or $a^2 = \lambda^2(h^2 + k^2 + l^2)/(4 \sin^2 \theta)$):

2θ	θ	$(\sin \theta)^2$	Ratio	(hkl)	a
6.19	3.095	0.00291	1.00	(111)	1.44
7.15	3.575	0.00389	1.33	(200)	1.44
10.12	5.06	0.00778	2.67	(220)	1.44
11.86	5.93	0.01067	3.66	(222)	1.51
12.4	6.2	0.01167	4.00	(311)	1.38

giving a lattice parameter of 1.44 nm if we average, but full marks will be given for indexing (assigning values of h , k and l to) any line and calculating a from that. [4]

3. The group velocity v of an electron in a crystal is related to its energy and wavevector by $v = \hbar^{-1} \partial E(k) / \partial k$. [2]

If an electron in any band is subjected to an electrical field, the resultant force will change its momentum, according to

$$\frac{\partial p}{\partial t} = \hbar \frac{\partial k}{\partial t} = -|e|\mathcal{E},$$

which can immediately be integrated to

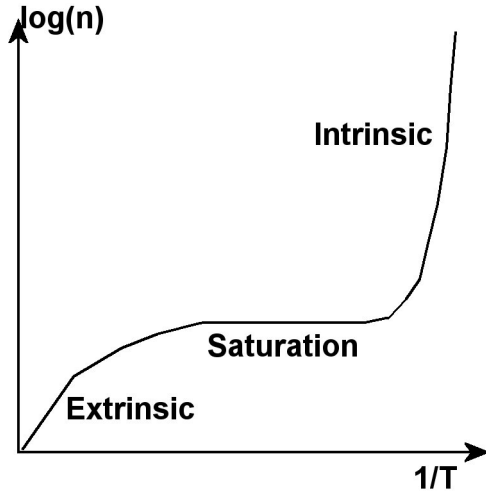
$$k(t) = k_0 - \frac{|e|\mathcal{E}t}{\hbar}.$$

So we see that the electron velocity is

$$v = \frac{2Aa}{\hbar} \sin \left(k_0 a - \frac{|e|\mathcal{E}a}{\hbar} t \right),$$

so that the velocity shows simple harmonic motion – the electron will oscillate backwards and forwards. (In reality, scattering will prevent full cycles from being observed, except in special structures) [5]

4. The variation of the number of carriers with temperature in a doped semiconductor is shown in the sketch below. It is important that the diagram should show the three regions (extrinsic, saturation and intrinsic, though they need not be specifically labelled). [4]



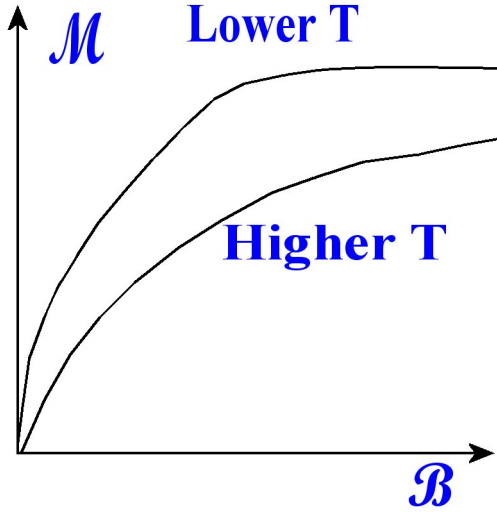
A Hall experiment would determine the sign of the majority carriers. [1]

We can assume that the dominant effect is the change in the number of carriers, so that (remember the exponential involves $E_g/2$, not E_g) [2]

$$\frac{\sigma(373)}{\sigma(273)} = \frac{\exp(-1.1 \cdot 1.6 \times 10^{-19} / (2 \cdot 373 \cdot 1.38 \times 10^{-23}))}{\exp(-1.1 \cdot 1.6 \times 10^{-19} / (2 \cdot 273 \cdot 1.38 \times 10^{-23}))} = 524.$$

Note that the exponential dependence dominates over the temperature dependence of the mobility, which varies here as $T^{-3/2}$, reducing the ratio by a factor of 0.63 – full marks for including only the exponential.

5. The variation of magnetisation \mathcal{M} as a function of applied field \mathcal{B} for an assembly of non-interacting spins at a fixed temperature is as shown below for two different temperatures (the important features are that the magnetisation saturates at high fields, and that this saturation is slower at higher temperature). [3]



At high temperatures we may write $\mathcal{M} = C\mathcal{B}/T$, with $\mathcal{B} = \mathcal{B}_{\text{ext}} + \lambda\mathcal{M}$ where \mathcal{B}_{ext} is the externally applied field. Hence

$$\mathcal{M} = C(\mathcal{B}_{\text{ext}} + \lambda\mathcal{M})/T$$

or

$$\mathcal{M} = \frac{C\mathcal{B}_{\text{ext}}}{T - \lambda C},$$

whence

$$\chi = \frac{\mathcal{M}}{\mathcal{H}_{\text{ext}}} = \frac{\mu_0 C}{T - \lambda C}. \quad [3]$$

6. If we take Maxwell's equation $\nabla \times \mathcal{H} = \mathcal{J}$, write $\mathcal{H} = \mathcal{B}/\mu_0$, take the curl of both sides, and substitute $\nabla \times \mathcal{J} = -\frac{nq^2}{m}\mathcal{B}$ we have

$$\nabla \times \nabla \times \mathcal{B} = \mu_0 \nabla \times \mathcal{J} = -\frac{\mu_0 n q^2}{m} \mathcal{B}.$$

Using $\nabla \times \nabla \times \mathcal{B} = \nabla(\nabla \cdot \mathcal{B}) - \nabla^2 \mathcal{B}$ and the fact that $\nabla \cdot \mathcal{B} = 0$ this gives

$$\nabla^2 \mathcal{B} = \frac{\mu_0 n q^2}{m} \mathcal{B}$$

which has solutions of the form $\mathcal{B} = \mathcal{B}_0 \exp(-\kappa x)$ with $\kappa^2 = \mu_0 n q^2 / m$. This represents an exponential decay, with a characteristic length, the inverse of κ , $\lambda = \sqrt{\frac{m}{\mu_0 n q^2}}$. [4]

If we assume that all the electrons in the system (of order 10^{28} m^{-3}) form pairs with charge $2e$ and mass $2m_e$ we have

$$\lambda \approx \sqrt{\frac{2 \cdot 9.11 \times 10^{-31}}{4\pi \times 10^{-7} \cdot 10^{28} (2 \cdot 1.6 \times 10^{-19})^2}} = 3.8 \times 10^{-8} \text{ m}.$$

Any result of order 10^{-7} to 10^{-8} m is acceptable. [3]